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Flux and potential reconstructions for guaranteed error bounds in numerical approximations of model PDEs

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Abstract. *We review how to bound the error between the unknown weak solution of a partial differential equation and its numerical approximation via a fully computable a posteriori estimate. We present a unified framework based on primal-conforming potential reconstructions, obtained via local Dirichlet finite element problems, and on dual-conforming equlilibrated flux reconstructions, obtained via local Neumann mixed finite element problems. The framework covers at once all standard numerical methods like the conforming, nonconforming, discontinuous Galerkin, and mixed finite elements. Model linear/nonlinear and steady/unsteady problems are considered, as well as the Laplace eigenvalue problem. We show how to achieve robustness both with respect to problem data such as the size of the nonlinearity or the final simulation time, as well with respect to the approximation polynomial degree. The derived estimates are valid on each iteration of a linearization procedure, as well as on each algebraic solver iteration. We use them to design fully adaptive algorithms including adaptive stopping and balancing criteria. Numerical experiments illustrate the theoretical developments.*